A Handbook for

Engineering Mathematics

Contains key theory concepts, formulae
and practice problems for

GATE

Also useful for ESE & other competitive examinations

MADE EASY
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Director’s Message

B. Singh (Ex. IES)

When the topic of completion of subjects comes while preparing for competitive exams, then studying one extra subject called as MATHEMATICS is often a tough pill to swallow. This is mainly due to the time constraints; as in this competitive environment when everybody is toiling, there is a lot to do in a limited time frame.

As it is rightly said,” Mathematics is not about numbers, equations, computations or algorithms it is about understanding.” Understanding mathematics is not as easy as it is said; to simplify this easy to say but difficult to be done task, the MADE EASY team has come up with this Handbook of Mathematics which contains all formulae and theoretical concepts of Engineering Mathematics.

And as we all know” the only way to learn mathematics is to do mathematics’’, so to facilitate all aspirants we have incorporated practice problems for GATE, which will help you to strengthen the concepts and gain confidence. This book will act as a two in one tool for preparation, initially will help in preparing the subject and later will serve as a revision aid with all formulae at one place.

B. Singh (Ex. IES)

CMD, MADE EASY Group

A Handbook for

Engineering Mathematics

CONTENTS

Unit-1: Basic Concepts 1-24

Unit-2: Calculus.. 25-55

Unit-3: Linear Algebra 56-88

Unit-4: Probability.. 89-131

Unit-5: Differential Equations 132-158

Unit-6: Vector Calculus 159-191

Unit-7: Complex Analysis 192-213

Unit-8: Numerical Methods 214-236

Unit-9: Laplace Transforms 237-242

Unit-10: Fourier Series 243-248

Unit-11: Partial Differential Equations (PDE) 249-257

Unit-12: Hypothesis Testing 258-267

A Handbook for
Engineering Mathematics

1

Basic Concepts

CONTENTS

I. Elementary Algebra 2

II. Geometry 5

III. Analytic Geometry 7

IV. Elementary Functions 13

V. Trigonometry 17

VI. Theory of Equations 23

1 Basic Concepts

I Elementary Algebra

1. Powers and Roots

(i) $a^0 = 1;\ a \neq 0$

(ii) $a^m a^n = a^{m+n}$

(iii) $\dfrac{a^m}{a^n} = a^{m-n}$

(iv) $(ab)^m = a^m b^m$

(v) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

(vi) $\left(a^m\right)^n = a^{mn}$

(vii) $a^{-m} = \dfrac{1}{a^m}$

(viii) $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

(ix) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(x) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

2. Logarithms

Definition: $y = \log_a(x)$ if and only if $a^y = x$ where $a, x > 0$ and $a \neq 1$.

Natural logarithm: $e^y = x$ if and only if $y = \log_e(x) = \ln(x)$

Where $e = \lim\limits_{x\to\infty}\left(1 + \dfrac{1}{x}\right)^x = 2.71828182846\ldots$

(i) $\log_a 1 = 0$

(ii) $\log_a a = 1$

(iii) $\log_a(mn) = \log_a m + \log_a n$

(iv) $\log_a\left(\dfrac{m}{n}\right) = \log_a m - \log_a n$

(v) $\log_a\left(m^n\right) = n\log_a m$

(vi) $\log_b a = \dfrac{1}{\log_a b}$

(vii) $\log_{(a^k)}(m) = \dfrac{1}{k}\log_a m$

(viii) $\log_a m = \log_b m \cdot \log_a b$ where $b > 0$ and $b \neq 1$

(ix) $\log_a m = \dfrac{\log_b m}{\log_b a}$

(x) $x^{\log_a y} = y^{\log_a x}$

(xi) $x = x^{\log_a a} = a^{\log_a x}$

(xii) $x = e^{\ln x} = \ln e^x$

3. Binomial Theorem

(i) Factorials

(a) $n! = 1\times2\times3\times\ldots\times(n-1)\times n$ (b) $0! = 1! = 1$

(ii) Binomial Coefficient $^{n}C_r = \dfrac{n!}{r!(n-r)!}$

(iii) Binomial Theorem

$$(x+y)^n = {}^{n}C_0x^n + {}^{n}C_1x^{n-1}y + {}^{n}C_2x^{n-2}y^2 + \ldots + {}^{n}C_ny^n$$

(iv) Product Formulas

(a) $(a+b)^2 = a^2 + 2ab + b^2$

(b) $(a-b)^2 = a^2 - 2ab + b^2$

(c) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(d) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

(v) Factoring Formulas

(a) $a^2 - b^2 = (a-b)(a+b)$

(b) $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

(c) $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$

(d) $a^{2n} - b^{2n} = \left(a^n - b^n\right)\left(a^n + b^n\right)$

(e) $a^n - b^n = (a-b)\left(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1}\right)$

Example: $\left(1-x^n\right) = (1-x)\left(1 + x + x^2 + x^3 + \ldots + x^{n-1}\right)$

(f) If n is odd then,

$$a^n + b^n = (a+b)\left(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots - ab^{n-2} + b^{n-1}\right)$$

Example:

(g) $a^5 - b^5 = (a-b)\left(a^4 + a^3b + a^2b^2 + ab^3 + b^4\right)$

(h) $a^5 + b^5 = (a+b)\left(a^4 - a^3b + a^2b^2 - ab^3 + b^4\right)$

4. Sequences

(i) Arithmetic sequence

$a, a+d,$ a+ $2d, a+3d, \ldots$

2

Calculus

CONTENTS

I.	Limit Continuity Differentiability	26
II.	Mean Value Theorems	28
III.	Taylor Series	29
IV.	Indeterminate Forms	33
V.	Maxima and Minima	36
VI.	Partial Derivatives	41
VII.	Total Differentiation	42
VIII.	Homogeneous Function	42
IX.	Maxima and Minima for Two Variable Functions	44
X.	Integration	46
XI.	Definite Integral	48
XII.	Length, Area, Volume	53

Calculus

2

I Limit Continuity Differentiability

1. Limit

$$\lim_{x\to a} f(x) = L \Leftrightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$$

Results

Assume $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist and c is any real number then

(i) $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$

(ii) $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

(iii) $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$

(iv) $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ provided $\lim_{x\to a} g(x) \neq 0$

(v) $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$

(vi) $\lim_{x\to a}\left[\sqrt[n]{f(x)}\right] = \sqrt[n]{\lim_{x\to a} f(x)}$

2. Continuity

A function $y = f(x)$ is continuous at $x = a$ if $\lim_{x\to a} f(x)$, $f(a)$ exists and

$\lim_{x\to a} f(x) = f(a)$

Otherwise, $f(x)$ is discontinuous at $x = a$.

3. Derivative

The derivative of $f(x)$ is defined as

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Provided the limit exists.

Interpretation of Derivative

(i) Geometrically, the value of $f'(x)$ represents the slope of the line tangent to the curve $y = f(x)$ at the point $(x, f(x))$

(ii) $f'(a)$ is the instantaneous rate of change of $f(x)$ at $x = a$.

(iii) If $f(x)$ is the position of an object than $f'(a)$ is the velocity of the object at $x = a$.

Results

(i) Polynomial function, sin x, cos x and e^x are continuous and differentiable for all x.

(ii) If $f(x)$ and $g(x)$ are continuous (differentiable) functions then

(a) $f(x) + g(x)$

(b) $f(x) - g(x)$

(c) $f(x) \cdot g(x)$

(d) $\frac{f(x)}{g(x)} (g \neq 0)$

(e) $fog\,(x) = f(g(x))$

are also continuous (differentiable).

(iii) Every differentiable function is continuous but every continuous function need not be differentiable.

4. Rules for Differentiation

(i) $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

(ii) $\frac{d}{dx}\left[a^x\right] = a^x \ln(a)$

(iii) $\frac{d}{dx}\left[e^x\right] = e^x$

(iv) $\frac{d}{dx}[\log_a x] = \frac{1}{x \ln(a)}$

(v) $\frac{d}{dx}[\ln x] = \frac{1}{x}$

(vi) $\frac{d}{dx}[\sin x] = \cos x$

(vii) $\frac{d}{dx}[\cos x] = -\sin x$

(viii) $\frac{d}{dx}[\tan x] = \sec^2 x$

(ix) $\frac{d}{dx}[\cot x] = -\csc^2 x$

(x) $\frac{d}{dx}[\sec x] = \sec x \tan x$

(xi) $\frac{d}{dx}[\csc x] = -\csc x \cot x$

(xii) $\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}}$

(xiii) $\frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$

(xiv) $\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$

(a) $f(x)$ is **NOT** differentiable at $x = 1$ for any values of a and b.
(b) $f(x)$ is differentiable at $x = 1$ for the unique values of a and b.
(c) $f(x)$ is differentiable at $x = 1$ for all the values of a and b such that $a + b = e$.
(d) $f(x)$ is differentiable at $x = 1$ for all values of a and b.

[EE-2017]

Answer Keys/Hints

1. (d)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (b)
6. (c)	**7.** (c)	**8.** (4)	**9.** $(-1/\sqrt{2})$	
10. (25)	**11.** (a)	**12.** (d)	**13.** (1)	
14. (90°)	**15.** (b)	**16.** (a)	**17.** (c)	**18.** (b)
19. (c)	**20.** (c)	**21.** (a)	**22.** (b)	

IV Indeterminate Forms

$$\left(\frac{0}{0}, \frac{\infty}{\infty}, 0.\infty, \infty - \infty, 1^{\infty}, \infty^{0}.\ 0^{0}\right)$$

L'Hospital's Rule

If $\dfrac{f(x)}{g(x)}$ has the indeterminate form $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$ and $g'(x) \neq 0$ for all $x \neq a$, then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Provide that $\displaystyle\lim_{x\to a}\frac{f'(x)}{g'(x)}$ exists or $\displaystyle\lim_{x\to a}\frac{f'(x)}{g'(x)} = \pm\infty$.

Standards Limit

(a) $\displaystyle\lim_{x\to 0}\frac{\sin x}{x} = 1$

(b) $\displaystyle\lim_{x\to 0}\frac{\tan x}{x} = 1$

(c) $\displaystyle\lim_{x\to 0}\frac{e^x - 1}{x} = 1$

(d) $\displaystyle\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$

(e) $\displaystyle\lim_{x\to a}\frac{x^n - a^n}{x - a} = na^{n-1}$

(f) $\displaystyle\lim_{x\to \infty}\frac{\sin x}{x} = 0$

(g) $\lim_{x \to 0} (1+x)^{1/x} = e$

(h) $\lim_{x \to 0} (1+ax)^{1/x} = e^a$

(i) $\lim_{x \to \infty} \left(1+\frac{1}{x}\right)^x = e$

(j) $\lim_{x \to \infty} \left(1+\frac{a}{x}\right)^x = e^a$

(k) $\lim_{x \to \infty} (x)^{1/x} = 1$

(l) $\lim_{x \to 0} (x)^x = 1$

Practice Problems

Find limits:

Q.1 $\lim_{x \to 0} \frac{x^3 + x^2}{2x^3 - 7x^2}$ **[2004]**

Q.2 $\lim_{x \to \infty} \frac{3x^2 - 4x + 5}{2x^2 + 3x + 2}$

Q.3 $\lim_{x \to \infty} \frac{x^{1/3} - 2}{x - 8}$ **[ME-2008]**

Q.4 $\lim_{n \to \infty} \left(1 - \frac{1}{n}\right)^{2n}$ **[CS-2010]**

Q.5 $\lim_{n \to \infty} \frac{n}{\sqrt{n^2 + n}}$ **[1999]**

Q.6 $\lim_{x \to 0} \frac{1}{10} \frac{1 - e^{-j5x}}{1 - e^{-jx}}$ **[IN-1999]**

Q.7 $\lim_{x \to \frac{\pi}{4}} \frac{\sin 2\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}}$ **[IN-2001]**

Q.8 $\lim_{x \to 0} x \sin \frac{1}{x}$

Q.9 $\lim_{x \to \infty} \frac{x - \sin x}{x + \cos x}$ **[ME-2014]**

Q.10 $\lim_{x \to 0} \frac{x - \sin x}{1 - \cos x}$ **[CS-2008]**

Q.11 $\lim_{x \to \infty} \left(\frac{x + \sin x}{x}\right)$ **[CE-2014]**

A Handbook for
Engineering Mathematics

3

Linear Algebra

CONTENTS

I. Matrices 57

II. Determinant 61

III. System of Linear Equations 69

IV. Eigen Values, Eigen Vectors 74

Linear Algebra

3

I Matrices

1. Matrix

Rectangular array of elements

$$A = [a_{ij}]_{mxn} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

2. Operations

Let $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$

Equal: $A = B$ if and only if $a_{ij} = b_{ij}\ \forall 1 \le i \le m,\ 1 \le j \le n$

Matrix addition:

$$A + B = [a_{ij}]_{m \times n} + [b_{ij}]_{m \times n} = [a_{ij} + b_{ij}]_{m \times n}$$

Scalar Multiplication: $cA = [ca_{ij}]_{\mathrm{m \times n}}$

Matrix multiplication:

If $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{n \times p}$ then $AB = [a_{ij}]_{m \times n}[b_{ij}]_{n \times p} = [c_{ij}]_{m \times p}$

Where $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} = a_{i1} b_{1j} + a_{i2} b_{2j} + \ldots + a_{in} b_{nj}$

3. Results

(i) Number of multiplications required to find $AB = m \times n \times p$

(ii) Number of additions required to find $AB = m \times (n - 1) \times p$

4. Transpose A^T

The transpose of a matrix is obtained by interchanging rows and columns of A.

Properties:

(i) $(A^{\mathrm{T}})^{\mathrm{T}} = A$

(ii) $(A + B)^{\mathrm{T}} = A^T + B^T$

(iii) $(cA)^T = c(A)^T$

(iv) $(AB)^T = B^T A^T$

5. Square matrix

$$A = [a_{ij}]_{m \times n} \quad \text{when } m = n$$

Trace of $A = [a_{ij}]_{n \times m}$: $Tr\,(A) = a_{11} + a_{22} + \ldots + a_{nn}$ = sum of main diagonal element

Upper Triangular matrix: A square matrix $[a_{ij}]_{n \times n}$ where $a_{ij} = 0$ if $i > j$.

Example: $\begin{bmatrix} a_{11} & a_{22} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$

Lower Triangular matrix: A square matrix $[a_{ij}]_{n \times n}$ where $a_{ij} = 0$ if $i < j$.

Diagonal matrix: A square matrix $[a_{ij}]_{n \times n}$ where $a_{ij} = 0$ if $i \neq j$.

Identity Matrix: An identity matrix is a square matrix $I_n = [a_{ij}]_{n \times n}$

where $a_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$

Results:

(i) $(AB)C = A(BC)$

(ii) $A^r \,.\, A^s = A^{r+s}$ $\quad r, s$: integers

(iii) $(A^r)^s = A^{rs}$

(iv) $D = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_n \end{bmatrix} \Rightarrow D^k = \begin{bmatrix} d_1^k & 0 & \cdots & 0 \\ 0 & d_2^k & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & d_n^k \end{bmatrix}$

Note: $AB \neq BA$

7. Invertible Matrix (Non Singular matrix)

$A_{n \times n}$ is invertible (or non-singular) if there exists a matrix $B_{n \times n}$

Such that $AB = BA = I_n$,

We write $B = A^{-1}$

A matrix that does not have an inverse is called **non-invertible** (or **singular**).

Answer Keys/Hints

1. $(a+1, 0)$	**2.** $(2 \pm i)$	**3.** (1, –1)	**4.** (4, 1)
5. (–1, –2, –3)	**6.** (0, 0, 3)	**7.** (2, 3, –2, 4)	
8. (1, 2, –2, –1)	**9.** –7.5	**10.** (49)	
11. $(x = -4, y = 10)$	**12.** (3, –5)	**13.** (3)	**14.** (6, –8)
15. (a)	**16.** (d)	**17.** (c)	**18.** (b)
19. (b)	**20.** (b)	**21.** (d)	**22.** (a)
23. (a)	**24.** (7, 10)	**25.** $(p-2)$	**26.** (1 & 25)
27. (1)	**28.** (3)	**29.** (c)	**30.** (0)
31. (5)	**32.** 1	**33.** (c)	**34.** (2)
35. (5)			

Practice Problems

Find Eigen vectors (1-4):

Q.1 $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

Q.2 $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

Q.3 $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Q.4 $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$

Q.5 The eigen vector (s) of the matrix $\begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\alpha \neq 0$ is (are)

[EC-1993]

Q.6 The number of linearly independent eigen vectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is